# Structural Study of the Low-Temperature Phase Transition in $Cs_3D(SeO_4)_2$

By M. Ichikawa

Department of Physics, Faculty of Science, Hokkaido University, Sapporo 060, Japan

AND T. GUSTAFSSON AND I. OLOVSSON

Institute of Chemistry, Uppsala University, Box 531, S-751 21 Uppsala, Sweden

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#### Abstract

The X-ray structure of tricaesium deuteriumbiselenate,  $Cs_3D(SeO_4)_2$ ,  $M_r = 686.7$ , has been studied at 25, 190 and 297 K with particular attention being paid to the low-temperature phase transition at  $T_{c3}$ = 180 K. The structure of  $Cs_3H(SeO_4)_2$  has also been studied at 297 K. The data were refined in the monoclinic space group C2/m, Z = 2, at 297 and 190 K, and in  $P2_1/m$ , Z = 2, at 25 K (Mo K $\alpha$  radiation,  $\lambda =$ 0.71073 Å). For  $Cs_3D(SeO_4)_2$ ,  $R(F^2) = 0.0365$  for 1801 unique reflections at 297 K,  $R(F^2) = 0.0389$  for 1787 unique reflections at 190 K and  $R(F^2) = 0.0405$ for 3514 unique reflections at 25 K. For  $Cs_3H(SeO_4)_2$ ,  $R(F^2) = 0.0637$  for 1265 unique reflections at 297 K. Temperature effects on the structure are mainly noticeable in the Cs-O bond distances of the oxygen coordination polyhedra of Cs (0.016 Å per 100 K on average). Accompanying the  $T_{c3}$  transition, the space group changes from C2/m to  $P2_1/m$ , and the two SeO<sub>4</sub> groups in the dimer become nonequivalent with one adopting  $HSeO_4^-$  character and the other  $SeO_4^{2-}$  character. As a result, the dimers have a net dipole moment and are arranged in an antipolar way, similar to  $K_3D(SO_4)_2$ . An examination of the room-temperature structure of  $Cs_3D(SeO_4)_2$  and other  $M_3H(XO_4)_2$ -type crystals reveals that the non-H atoms lie in approximately the same position in both cases and that the only major difference is that half of the hydrogens in other  $M_3H(XO_4)_2$ -type crystals are involved in hydrogen-bonded dimers which are formed with two different adjacent selenate groups. The successive transitions in  $Cs_3D(SeO_4)_2$  are characterized as an order-disorder transition of the donor-oxygen atom at  $T_{c1}$ , reorganization of the hydrogen bonds at  $T_{c2}$  and an order-disorder transition of the proton at  $T_{c3}$ .

#### 1. Introduction

Tricaesium hydrogenbiselenate  $Cs_3H(SeO_4)_2$  (abbreviated to TCHSe) and its deuterated analogue

 $Cs_3D(SeO_4)_2$  (TCDSe) are members of a family of hydrogen-bonded compounds with the general formula  $M_3H(XO_4)_2$  (M = K, Rb, Cs, and X = S, Se), the crystal structures of which are presently being systematically studied. For further information on the characteristics of, and interest in, these compounds, see Ichikawa, Gustafsson & Olovsson (1992). All  $M_3H(XO_4)_2$ -type crystals have the same trigonal ( $R\bar{3}m$ ) superionic and paraelastic phase (I) above the transition temperature  $T_{c1}$  (range 339– 456 K). TCHSe and TCDSe, however, are unique in the following respects:

(1) TCHSe and TCDSe have an extra intermediate ferroelastic phase (II) above  $T_{c2}$ , between the room-temperature (RT) ferroelastic phase (III) and the high-temperature phase (I) (Komukae, Osaka, Kaneko & Makita, 1985).

(2) While the space group of the RT phase of all other members is the same (A2/a), recent optical and structural studies reveal that the space group of TCHSe in the RT phase (III) is C2/m (Merinov, Baranov, Tregubchenko & Shuvalov, 1988; Merinov, Bolotina, Baranov & Shuvalov, 1988).

(3) Quite recently the intermediate phase (II) has been shown to be monoclinic (A2/a), the same as the RT phase of other members (Merinov, Baranov & Shuvalov, 1990).

(4) The transition temperature  $T_{c3}$  of TCDSe at 180 K, between the RT phase (III) and the low-temperature (LT) phase (IV), is the highest among the members (cf. 20–90 K for the other members) and it falls in the normal temperature range observed in hydrogen-bonded ferroelectric crystals such as  $KH_2PO_4$  (Komukae *et al.*, 1985; Ichikawa, Gustafsson & Olovsson, 1991).

In order to understand the nature of the successive phase transitions in  $M_3H(XO_4)_2$ -type crystals, we considered it to be of great interest to investigate the similarities and differences between the Cs<sub>3</sub>H(SeO<sub>4</sub>)<sub>2</sub> [and Cs<sub>3</sub>D(SeO<sub>4</sub>)<sub>2</sub>] crystal and the other members, and also between the different phases of Cs<sub>3</sub>H(SeO<sub>4</sub>)<sub>2</sub> [Cs<sub>3</sub>D(SeO<sub>4</sub>)<sub>2</sub>], since their mutual relationship does not seem to have been fully explored. To this end, in

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particular focusing on the LT phase transition at  $T_{c3}$ , the X-ray structure of Cs<sub>3</sub>D(SeO<sub>4</sub>)<sub>2</sub> has been investigated at 297 and 190 K (above  $T_{c3}$ ) and at 25 K (below  $T_{c3}$ ), together with that of Cs<sub>3</sub>H(SeO<sub>4</sub>)<sub>2</sub> at 297 K.

## 2. Experimental

Single crystals of  $Cs_3H(SeO_4)_2$  were prepared by cooling, from around 313 K, a saturated aqueous solution containing excess selenic acid. A specimen of dimensions  $0.11 \times 0.06 \times 0.10$  mm was cut from a ferroelastic single-domain part of the batch. The lattice constants are from Ichikawa & Sato (1987) and are given in Table 1. The preparation of  $Cs_3D(SeO_4)_2$  single crystals is described by Ichikawa *et al.* (1991) and the specimen used was a hexagonal plate with a ferroelastic single domain and with a width of 0.25 mm and thickness of 0.06 mm; the unit-cell parameters are taken from Ichikawa *et al.* (1991) and are given in Table 1.

The data were collected using Mo  $K\alpha$  radiation (0.71073 Å) and a Huber four-circle X-ray diffractometer with a closed-cycle helium refrigerator (Samson, Goldish & Dick, 1980). The temperature was measured by silicon sensors installed in the cold-station extender  $(T_A)$  and in the  $\varphi$  shaft of the diffractometer  $(T_B)$ . The temperature at the specimen  $(T_S)$  was calibrated against the transition temperatures of KH<sub>2</sub>PO<sub>4</sub> (123 K) and KMnF<sub>3</sub> (81 and 187 K). The stability of the temperature during the measurement was within 1 K. The accuracy of the temperature of the crystal is estimated to be within 2 K.

Data collection was carried out at 297 K for  $Cs_3H(SeO_4)_2$ . Intensity measurements were carried out in the  $\omega$ -2 $\theta$  scan mode. The step width was  $0.015^{\circ}$  in  $\omega$  with a minimum number of 70 steps plus the  $\alpha_1 - \alpha_2$  splitting. The measuring time varied from 0.5 to 2.0 s step<sup>-1</sup> in the range  $2 < 2\theta < 70^{\circ}$  for  $-17 \le h \le 14$ ,  $-10 \le k \le 0$ ,  $-12 \le l \le 13$  (max.  $\sin\theta/\lambda = 0.8069 \text{ Å}^{-1}$ ). For Cs<sub>3</sub>D(SeO<sub>4</sub>)<sub>2</sub> the data collection was carried out in the order 190, 25 and 297 K. Intensity measurements were carried out in the  $\omega$ -2 $\theta$  scan mode. The step width was 0.010° with a minimum number of 74 steps plus the  $\alpha_1 - \alpha_2$ splitting. The measuring time varied from 0.3 to 2.0 s step<sup>-1</sup> in the range  $2 < 2\theta < 80^\circ$  for  $-19 \le h \le$ 0,  $-11 \le k \le 11$ ,  $-14 \le l \le 15$  (max.  $\sin \theta / \lambda =$ 0.9044  $Å^{-1}$ ). Five standard reflections monitored every 3h were used to check the stability of the measurements. The intensities and their standard deviations were corrected for variations by the method of McCandlish, Stout & Andrews (1975). For TCHSe, the correction factors were 1.0-0.958 and for TCDSe, 1.0-0.985 at 297 K, 1.0-1.012 at 190 K, 1.0–1.041 at 25 K. The data set was corrected

### Table 1. Crystallographic data

The unit-cell dimensions were determined in the papers cited below, although no values are quoted there. Monoclinic, Z = 2, F(000) = 596. Cs<sub>3</sub>D(SeO<sub>4</sub>)<sub>2</sub>, Mo K $\alpha_1$  ( $\lambda = 0.70930$  Å); Cs<sub>3</sub>H(SeO<sub>4</sub>)<sub>2</sub>, Ag K $\alpha$  ( $\lambda = 0.56087$  Å).

		$Cs_3D(SeO_4)_2$	· C	Cs <sub>3</sub> H(SeO <sub>4</sub> ) <sub>2</sub> †		
	25 K	190 K	297 K	295 K		
Number and	26 (55-59)	26 (54-59)	26 (54-58)	40 (20-22)		
$2\theta$ range (°) of reflections used						
a (Å)	10.7906 (11)	10.8465 (7)	10.8911 (9)	10.8921 (15)		
b (Å)	6.3264 (3)	6.3551 (2)	6.3830 (3)	6.3864 (9)		
c (Å)	8.3669 (20)	8.4040 (12)	8.4483 (16)	8.4441 (12)		
β(°)	112.215 (27)	112.385 (16)	112.454 (21)	112.422 (12)		
V (Å <sup>3</sup> )	528.78 (22)	535.64 (13)	542.42 (17)	542.98 (12)		
$D_x (g \text{ cm}^{-3})$	4.312	4.257	4.204	4.200		
* Ichikawa et al. (1991).						
† Ichikawa & Sato (1987).						

for background effects (Lehmann & Larsen, 1974) and Lorentz, polarization and absorption effects (for TCHSe, transmission factors 0.2654–0.4405, for TCDSe, 0.0475–0.3176 at 297 K, 0.0452–0.3138 at 190 K, 0.0440–0.3110 at 25 K). Other experimental details are summarized in Table 2.

The space group of Cs<sub>3</sub>D(SeO<sub>4</sub>)<sub>2</sub> is C2/m above the LT transition at  $T_{c3}$  [the same as Cs<sub>3</sub>H(SeO<sub>4</sub>)<sub>2</sub> (Merinov, Bolotina *et al.*, 1988)] and P2<sub>1</sub>/m or P2<sub>1</sub> below  $T_{c3}$  (Ichikawa *et al.*, 1991). In order to check the possibility of cell doubling below the  $T_{c3}$  transition, a Q scan was carried out at 25 K along the  $a^*$ ,  $b^*$  and  $c^*$  axes and [hh0], [0kk], [h0h] and [hhh] directions. No evidence of cell doubling or tripling was found.

The refinement (on  $F^2$ ) of both the TCHSe and TCDSe data started with the RT TCHSe structure reported by Merinov, Bolotina et al. (1988). All atoms except hydrogen were refined with anisotropic displacement parameters; the H atom with an isotropic displacement parameter. For the 25 K data, refinement was performed in the space group  $P2_1/m$ , with a fixed isotropic displacement parameter for D. The residual peaks in the final  $\Delta \rho$  map were larger in this case compared to the other three data sets [the largest positive and negative peaks are seen around Cs(1)]. This might indicate a small degree of disorder and could be the reason for the larger wR value, but detailed examination is left for the future. Refinement was also carried out in the alternative space group  $P2_1$ , but no significant improvement was achieved.

A type-(I) isotropic extinction parameter with Lorentzian distribution resulted in the best agreement between observed and calculated data for the four data sets. Some reflections were omitted at the later stages of the refinement (see Table 2). Details of the refinements are given in Table 2.

All calculations were performed using the program system described by Lundgren (1983) and the struc-

		$Cs_3D(SeO_4)_2$		$Cs_3H(SeO_4)_2$
	25 K	190 K	297 K	297 K
$\mu (mm^{-1})$	17.04	16.82	16.60	16.60
Total number of reflections measured and used	7279	3478	3507	1351
Number of unique reflections	3514	1787	1801	1265
R <sub>int</sub>	0.0194	0.0259	0.0225	0.0322
Final least-squares refinement				
Function minimized	$\sum w(F_o^2 - F_c^2)^2$	$\sum w(F_o^2 - F_c^2)^2$	$\sum w(F_o^2 - F_c^2)^2$	$\sum w(F_o^2 - F_c^2)^2$
Weights	$1/\sigma^2(F^2)$	$1/\sigma^2(F^2)$	$1/\sigma^2(F^2)$	$1/\sigma^2(F^2)$
Excluded reflections	1* + 12†	2* + 15†	2 <b>*</b> + 6†	1*+2†
Number of parameters	77	40	40	40
Isotropic secondary- extinction parameter	0.18 (1) × 10 <sup>4</sup>	0.53 (1) × 10 <sup>4</sup>	$0.96(6) \times 10^3$	$0.30(1) \times 10^4$
$R(F^2)$	0.0405	0.0389	0.0365	0.0637
wR	0.0877	0.0484	0.0582	0.0733
S	3.60	1.76	1.94	2.22
Max. $\Delta/\sigma$	0.02	0.003	0.02	0.05
Final $\Delta \rho$ map				
Max. and min. $\Delta \rho$ (e Å <sup>-3</sup> )	5.94, -6.52	1.43, -1.39	1.61, -0.67	1.44, -2.57
	* Extinction	1.		
	† Multiple 1	eflection.		

### Table 2. Experimental details

ture was drawn using ORTEPII (Johnson, 1976). Atomic scattering factors and mass attenuation coefficients were taken from *International Tables for* X-ray Crystallography (1974, Vol. IV). Table 3. Fractional coordinates and equivalent iso-<br/>tropic displacement parameters (Ų)The equivalent isotropic displacement factor is of the form:

### 3. Results and discussion

Final positional and (equivalent) isotropic displacement parameters at 297, 190 and 25 K are given in Table 3.\* The projection of the structure along b is shown in Fig. 1. Temperature dependencies of selected interatomic distances and angles are given in Table 4.

## 3.1. General features of the structure

The structure of  $Cs_3D(SeO_4)_2$ , which is isomorphous with  $Cs_3H(SeO_4)_2$ , is built up of hydrogenbonded SeO<sub>4</sub> dimers and Cs—O polyhedra (Fig. 1). There are two kinds of Cs atoms: Cs(1) occupies a special position on a twofold axis; Cs(2) lies in a general position. Both types of Cs atoms are surrounded by ten O atoms, as in other  $M_3H(XO_4)_2$ -type crystals. The Cs(1)—O bond lengths are shorter than the Cs(2)—O bond lengths by about 0.02 Å on average (0.021 Å for TCHSe and 0.018 Å for TCDSe). This trend is analogous to that observed in Rb<sub>3</sub>D(SeO<sub>4</sub>)<sub>2</sub> (Ichikawa *et al.*, 1992), although less prominent in the Cs compound. The two SeO<sub>4</sub><sup>2-</sup> ions

$\boldsymbol{B}_{eq} = (4/3) \sum_{i} \sum_{j} \boldsymbol{\beta}_{ij} (\mathbf{a}_{i} \cdot \mathbf{a}_{j}).$					
H(SeO₄)	x	У	z	$B_{eq}(B)$	

			_	~ ea(~)
Cs <sub>3</sub> H(SeO) 297 K	a) <sub>2</sub>	ž		uų (
Cs(1)	0	0	0	2.16
Cs(2)	0.19761 (3)	0	0.60724 (5)	2.30
Se	0.42702 (5)	0	0.23495 (7)	1.65
O(1)	0.48639 (25)	0.21103 (42)	0.69056 (32)	2.96
O(2)	0.39058 (42)	0	0.02072 (48)	2.59
O(3)	0.28813 (37)	0	0.26199 (58)	3.05
Н	0.5	0	0	8.7 (3.9)
Cs <sub>3</sub> D(SeO	4)2			
297 K				
Cs(1)	0	0	0	2.20
Cs(2)	0.19761 (1)	0	0.60725 (2)	2.34
Se	0.42686 (2)	0	0.23473 (3)	1.80
O(1)	0.48646 (12)	0.21134 (15)	0.69104 (15)	2.99
O(2)	0.39049 (20)	0	0.02109 (20)	2.79
O(3)	0.28748 (16)	0	0.26264 (26)	3.21
D	0.5	0	0	9.7 (2.4)
190 K				
Cs(1)	0(1)	0	0	1.35
Cs(2)	0.19775 (1)	0	0.60691 (1)	1.38
Se	0.42695 (1)	0	0.23541 (2)	0.98
O(1)	0.48589 (8)	0.21251 (11)	0.69003 (11)	1.82
O(2)	0.39012 (14)	0	0.02034 (14)	1.69
O(3)	0.28684 (12)	0	0.26336 (18)	1.90
D	0.5	0	0	5.2 (1.4)
25 K				
Cs(1)	0.25909 (2)	0.25	- 0.00105 (2)	0.68
Cs(21)	0.44858 (2)	0.25	0.60871 (2)	0.61
Se(1)	0.67699 (3)	0.25	0.23682 (4)	0.69
O(11)	0.76588 (13)	0.46352 (19)	0.31393 (18)	0.45
O(21)	0.64006 (20)	0.25	0.02292 (26)	0.67
O(31)	0.53877 (19)	0.25	0.27255 (29)	0.88
D	0.782 (6)	0.25	- 0.012 (10)	15
Cs(22)	0.05256 (2)	0.25	- 0.60372 (2)	0.59
Se(2)	-0.17527 (3)	0.25	-0.23536 (4)	0.70
O(12)	-0.26293 (13)	0.46441 (19)	-0.30645 (18)	0.69
O(22)	-0.13602 (20)	0.25	-0.01625 (26)	0.64
O(32)	-0.03346 (19)	0.25	-0.26030 (28)	0.74

<sup>\*</sup> Lists of anisotropic displacement parameters, interatomic distances and angles, structure factors and a *c*-axis projection of the structure have been deposited with the British Library Document Supply Centre as Supplementary Publication No. SUP 55127 (78 pp.). Copies may be obtained through The Technical Editor, International Union of Crystallography, 5 Abbey Square, Chester CH1 2HU, England. [CIF reference: AB0279]

in the hydrogen-bonded dimers are related by a twofold axis, with hydrogen either on the twofold axis or in two disordered positions about it.

The systematic trends in the bond distances and angles of the SeO<sub>4</sub> tetrahedra are analogous to those in Rb<sub>3</sub>D(SeO<sub>4</sub>)<sub>2</sub> (Ichikawa *et al.*, 1992); for a detailed discussion this paper should be consulted. It is noted that at RT the O(2)—O(3) distance is larger than the O(1)—O(2) distance and rather close to other O—O distances involving O atoms without an attached proton, in contrast to the regular HSeO<sub>4</sub><sup>-</sup> group. This is because the O(2)—Se—O(3) angle is considerably larger than the O(1)—Se—O(2) angle (Table 4). This trend is also observed in the case of Rb<sub>3</sub>D(SeO<sub>4</sub>)<sub>2</sub>.

The trend in the equivalent isotropic displacement parameters  $B_{eq}$  which was pointed out for the case of Rb<sub>3</sub>D(SeO<sub>4</sub>)<sub>2</sub> is also observed in the present results:  $B_{eq}(Se) < B_{eq}(Cs) < B_{eq}(O)$  (except at 25 K). The relation  $B_{eq}[Cs(1)] < B_{eq}[Cs(2)]$  is also noted. This trend is also consistent with the observation that  $[Cs(1)-O]_{av} < [Cs(2)-O]_{av}$  (details are given in Table 6, deposited). These relations are characteristic of all  $M_3H(XO_4)_2$ -type crystals so far studied. The disk-shaped anisotropy of the displacement parameters for O(2) and O(3) is also analogous to that observed for Rb<sub>3</sub>D(SeO<sub>4</sub>)<sub>2</sub> (Ichikawa *et al.*, 1992).



Fig. 1. The *b*-axis projection of the RT structure of  $Cs_3H(SeO_4)_2$ . Displacement ellipsoids are scaled to include 50% probability. The isotropic displacement parameter for hydrogen is arbitrarily given the value 4.0 Å<sup>2</sup>.

# 3.2. Comparison between the room-temperature results

The present results for  $Cs_3H(SeO_4)_2$  at 297 K agree well with those at 295 K obtained by Merinov, Bolotina *et al.* (1988): the differences in the bond distances and angles are generally  $\leq \sigma$  with a maximum value of  $1.7\sigma$ .

In order to study the isotope effect a comparison was also made between the bond distances and angles of Cs<sub>3</sub>D(SeO<sub>4</sub>)<sub>2</sub> at 297 K and those of Cs<sub>3</sub>H(SeO<sub>4</sub>)<sub>2</sub> at 297 K and at 295 K as observed by Merinov, Bolotina et al. (1988). Our TCDSe and TCHSe data gave differences of less than  $2.3\sigma$  (a detailed comparison is made in Table 6, deposited). The differences may be due to errors in the results since the effects on the distances and angles, other than on the hydrogen-bonded distance, are only indirect and should be very small. These results are consistent with the lattice-constant results, where no significant isotope effect could be found (Ichikawa et al., 1991). The hydrogen-bonded distance O(2)··· $O(2^{ii})$  at RT in TCHS is 2.538 (8) Å (this work) and 2.535 (9) Å (Merinov, Bolotina et al., 1988), which is shorter than that in TCDSe [2.543 (4) Å]. This trend agrees with earlier empirical results (Olovsson & Jönsson, 1976; Ichikawa, 1978). However, the isotope effect, 0.005 (9) Å (our data) and 0.008 (10) Å [comparison with the data of Merinov, Bolotina et al. (1988)] is hardly significant compared to the standard deviations.

# 3.3. Temperature dependence and phase transition at $T_{c3}$

Temperature changes in the structure are mainly noticeable in the Cs—O bond distances of the oxygen coordination polyhedra of Cs (0.016 Å per 100 K on average) [cf. 0.015 Å per 100 K for Rb—O in Rb<sub>3</sub>D(SeO<sub>4</sub>)<sub>2</sub> on average (Ichikawa *et al.*, 1992)]. These changes may explain the temperature dependence of the lattice constants. No noticeable temperature dependence could be seen for the other bond distances and angles.

The space group changes from C2/m to  $P2_1/m$  on passing through the  $T_{c3}$  transition with decreasing temperature. The structural changes accompanying the  $T_{c3}$  transition are clearly in contrast to  $Rb_3D(SeO_4)_2$ , but are similar to those in  $K_3D(SO_4)_2$ where breaking of the A-centering is clearly seen and the space group changes from A2/a to  $P2_1/a$  (Noda, Watanabe, Kasatani, Terauchi & Gesi, 1991). The atoms located in the mirror plane in the RT phase (III) are still in the mirror plane, but as a result of the disappearance of the twofold axis, the Cs(1) atom no longer lies on a twofold axis and Cs(2) splits into Cs(21) and Cs(22). The D atom, which was on the twofold axis (or more plausibly, occupied two

### Table 4. Selected bond distances (Å) and angles (°)

Bond distances have not been corrected for thermal motion. The corresponding values for  $Cs_3H(SeO_4)_2$  at 295 K obtained by Merinov, Bolotina *et al.* (1988) are also given for comparison. Symmetry code: (i) 1 + x, y, z; (ii) 1 - x, -y, -z.

	$Cs_3D(SeO_4)_2$				$Cs_3H(SeO_4)_2$	
	25 K		190 K	297 K	297 K	295 K
SeO <sub>4</sub> ion Se(1)—O(11) (× 2) —O(21) —O(31) Average	1.641 (2) 1.679 (1) 1.628 (2) 1.647		1.631 (1) 1.696 (1) 1.624 (1) 1.645	1.629 (1) 1.694 (2) 1.623 (2) 1.643	1.628 (3) 1.698 (4) 1.614 (4) 1.642	1.628 (4) 1.703 (5) 1.623 (6) 1.646
Se(2)O(12) (×2) O(22) O(32) Average	1.631 (2) 1.719 (3) 1.621 (2) 1.651					
$\begin{array}{c} O(11) - Se(1) - O(11) \\ - O(21) & (\times 2) \\ - O(31) & (\times 2) \\ O(21) - Se(1) - O(31) \\ Average \end{array}$	110.8 (1) 106.7 (1) 111.6 (1) 109.3 (1) 109.5	$\begin{array}{c} O(1) - Se(1) - O(1) \\ - O(2) & (\times 2) \\ - O(3) & (\times 2) \\ O(2) - Se(1) - O(3) \end{array}$	111.8 (1) 106.0 (1) 112.5 (1) 107.5 (1) 109.4	111.9 (1) 106.0 (1) 112.4 (1) 107.7 (1) 109.4	111.8 (2) 106.1 (1) 112.4 (1) 107.5 (2) 109.4	112.0 (2) 106.1 (2) 112.5 (2) 107.0 (2) 109.4
O(12)—Se(2)—O(12) —O(22) (× 2) —O(32) (× 2) O(22)—Se(1)—O(32) Average	112.4 (1) 105.3 (1) 113.5 (1) 105.8 (1) 109.3					
O(11)O(11) -O(21) (×2) -O(31) (×2) O(21)-O(31) Average	2.701 (3) 2.664 (3) 2.704 (3) 2.697 (4) 2.689	$\begin{array}{c} O(1) - O(1) \\ - O(2) & (\times 2) \\ - O(3) & (\times 2) \\ O(2) - O(3) \end{array}$	2.701 (1) 2.658 (1) 2.705 (1) 2.677 (2) 2.684	2.698 (2) 2.653 (2) 2.702 (2) 2.678 (3) 2.681	2.695 (5) 2.658 (4) 2.694 (4) 2.670 (6) 2.678	2.698 (5) 2.663 (6) 2.704 (6) 2.675 (8) 2.685
O(12)O(12) -O(22) (× 2) -O(32) (× 2) O(22)O(32) Average	2.712 (3) 2.664 (3) 2.720 (2) 2.664 (4) 2.691					
Hydrogen bond $O(21)\cdots O(22^{i})$	2.558 (4)	O(2)…O(2")	2.534 (3)	2.543 (4)	2.538 (8)	2.535 (9)

disordered positions close to the twofold axis), shifts along the O(2)···O(2<sup>ii</sup>) hydrogen bond. Thus Se(1)O<sub>4</sub> in the  $H(SeO_4)_2^{3-}$  ion becomes closer to an  $SeO_4^{2-}$ ion and  $Se(2)O_4$  becomes closer to an  $HSeO_4^-$  ion. Table 4 clearly shows that the bond distances and angles in Se(1)O<sub>4</sub> shift in such a way as to reduce the distortion and those in Se(2)O<sub>4</sub> shift so as to increase the distortion. As a result the dimer has a net dipole moment, since the deviation of the Se atom from the center of the O atoms is larger in HSeO<sub>4</sub> than in SeO<sub>4</sub> (Ichikawa, 1987, 1988). The dipole moment vectors lie in the mirror plane, but they are arranged antiparallel along the *a* axis, parallel along the *b* axis, and antiparallel to each other between two adjacent arrays of the dipole-moment vectors in the (100) plane. As a whole the LT structure is antipolar and could be antiferroelectric. It is, however, still uncertain whether the LT phase really is antiferroelectric, since neither a ferroelectric nor an antiferroelectric hysteresis loop has been observed, not only in Cs<sub>3</sub>D(SeO<sub>4</sub>)<sub>2</sub> but also in other  $M_3H(XO_4)_2$ -type crystals (Gesi, 1980; Komukae et al., 1985; Baranov, Merinov, Tregubchenko, Shuvalov & Shchagina, 1988). If we compare the primitive cell of the LT phase (IV) and the RT phase (III), a cell doubling occurs in the LT phase (IV). This is similar to  $K_3D(SO_4)_2$  (Noda *et al.*, 1991).

# 3.4. Relation between room-temperature structures of $Cs_3H(SeO_4)_2$ and other $M_3H(XO_4)_2$ -type crystals

The RT structures of  $Cs_3H(SeO_4)_2$  and other  $M_3H(XO_4)_2$ -type crystals are similar in certain respects, but clearly differ in other respects. Although the mutual relation of their unit cells was discussed by Ichikawa et al. (1991), the relationship between the atomic arrangements does not seem to have been fully clarified. For this reason, the corresponding projections for both types of structure have been examined. In Fig. 2(a) the projection of the RT structure along  $-\frac{1}{2}\mathbf{b} + \frac{1}{2}\mathbf{c}$  is given for  $Rb_3D(SeO_4)_2$ ; this corresponds to the *c*-axis projection of  $Cs_3D(SeO_4)_2$ , which is shown in Fig. 2(b). It is seen that the atomic positions for non-H atoms are approximately the same in both structures (cf. also Fig. 4, deposited). The main difference is that half of the H atoms in the Rb compound are involved in hydrogen-bonded dimers which are formed with two different adjacent selenate groups; this breaks the Cbase centering which is valid for TCHSe (Fig. 2).

The symmetry of phase (II) of TCHSe was first suggested to be either monoclinic or triclinic by Baranov, Tregubchenko, Shuvalov & Shchagina (1987). Subsequently, Merinov, Baranov *et al.* (1988) concluded that it is triclinic, taking into account the orientation of the domain boundaries as shown using a polarizing microscope. They later confirmed this by an X-ray study. However, quite recently Merinov *et al.* (1990) have examined the data obtained by Komukae and co-workers further (reference 9 in Merinov *et al.*, 1990) and reached the conclusion that the space group of phase (II) is monoclinic, A2/a, the



Fig. 2. (a) Projection of the RT structure of  $Rb_3D(SeO_4)_2$  along the  $-\frac{1}{2}b + \frac{1}{2}c$  direction. The solid lines denote the unit cell and the dotted lines correspond to the unit-cell axes of  $Cs_3H(SeO_4)_2$ projected along the c axis. (b) The RT structure of  $Cs_3H(SeO_4)_2$ projected along the c axis.

same as in other  $M_3H(XO_4)_2$ -type crystals at RT. The mutual relationship of all the phases is shown in Fig. 3. [Fig. 2 in Ichikawa *et al.* (1991) should be corrected.]

### 3.5. Atomic movement through successive phase transitions

Based on the above discussion, the atomic motions accompanying successive phase transitions can be described without a detailed knowledge of the structure of phase (II). Combining the present results with earlier work, the following atomic motions occur at the transitions.

In the superionic phase (I), the O(2) atom is dynamically disordered with a population of  $\frac{1}{3}$  in

Fig. 3. Mutual relationship of the unit cells in each phase of  $Cs_3H(SeO_4)_2$ . Thick solid lines denote the hexagonal  $R^3m$  cell of phase (I), broken lines the monoclinic A2/a cell of phase (II), and thin solid lines the monoclinic C2/m cell of phase (III). The symbols h, mA and mC denote hexagonal, monoclinic A base-centered, and monoclinic C base-centered cells, respectively. Note that the unit cell in phase (IV) is the same as in phase (III), but primitive (see text). The transformation matrices from phase (I) to phases (II) and (III) are

$$\begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & 0 \\ -\frac{2}{3} & -\frac{1}{3} & \frac{3}{2} \end{pmatrix} \text{ and } \begin{pmatrix} 1 & 2 & 0 \\ -1 & 0 & 0 \\ -\frac{1}{3} & \frac{2}{3} & \frac{1}{3} \end{pmatrix},$$

respectively. The space groups of phase (I)  $(R\overline{3}m)$ , phase (III) (C2/m) and phase (IV)  $(P2_1/m)$  have the relationship of group/ subgroup, but the space group of phase (II) (A2/a) bears no such relation to the others (Merinov, Bolotina *et al.*, 1988; Merinov, Baranov *et al.*, 1988). three equivalent positions around the hexagonal c axis (Baranov, Makarova, Muradyan, Tregubchenko, Shuvalov & Simonov, 1987; Merinov *et al.*, 1990). This implies that the H atom also jumps from one site to another, following the O(2) atom, *i.e.* one hydrogen bond breaks and another is formed. The transition at  $T_{c1}$  can thus can be characterized as an order-disorder transition not only of hydrogen but also of the donor-oxygen atom in the hydrogenbonded system.

In phase (II), where as stated earlier the space group is the same (A2/a) as that in the RT phase of the other members, all H atoms are disordered around the center of symmetry (see Fig. 2a). At the  $T_{c2}$  transition, half of the H atoms break hydrogen bonds and form alternative hydrogen bonds so as to form a dimer between two different adjacent selenate groups. As a result of this, non-H atoms shift their positions slightly, all atoms except O(1) lie in a mirror plane, and H atoms become disordered around a twofold axis in phase (III) (see Fig. 1). On passing through  $T_{c3}$ , the proton becomes ordered, as a result of which the twofold axis and the C base centering disappear and the primitive cell becomes doubled in phase (IV).

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# Electron Difference Density and Vibration Tensors in SrTiO<sub>3</sub>

BY R. H. BUTTNER AND E. N. MASLEN

Department of Physics, University of Western Australia, Nedlands 6009, Australia

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### Abstract

Multiple data sets for strontium titanate SrTiO<sub>3</sub>:  $M_r = 183.51$ , cubic,  $Pm\bar{3}m$ , a = 3.9092 (4) Å, V = 59.740 (3) Å<sup>3</sup>, Z = 1,  $D_x = 5.101$  Mg m<sup>-3</sup>,  $\lambda$ (Mo K $\alpha$ ) = 0.71069 Å,  $\mu = 26.489$  mm<sup>-1</sup>, F(000) = 84, T = 298 K, refinement indices range R = 0.013-0.028, wR = 0.012-0.028 for 91 unique reflections. Seven structure refinements from five different flux-grown crystals of SrTiO<sub>3</sub> are compared. The specimens selected showed mild to modest extinction. In all cases the mean-square vibration amplitude for the Ti atom along the Ti-O bond markedly exceeds that for the O atom. The amplitudes of the cations are

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